STAT-DSE-2(A)

Operational Research

UNIT II

Assignment Problem

ASSIGNMENT PROBLEM

….. O.R. Produces an integrated solution for the good of entire organization and not just to effect local environment…..

INTRODUCTION

- The assignment problem is a special case of transportation in which the objective is to assign a number of resources to the equal number of activities at a minimum cost.(or maximum profit).
- The GOAL of a general assignment problem is to find an optimal assignment of machines (laborers) to jobs without assigning an agent more than once and ensuring that all jobs are completed. The objective might be to minimize the total time to complete a set of jobs, or to maximize skill ratings, maximize the total satisfaction of the group or to minimize the cost of the assignments.

STRUCTURE OF ASSIGNMENT PROBLEM.

As mentioned earlier, assignment problem is a special type of transportation problem in which

1. Number of supply and demand nodes are equal.
2. Supply from every supply node is one.
3. Every demand node has a demand of one.
4. Solution is required to be all integers.

HUNGARIAN METHOD

- Also known as ,reduced matrix method
- And efficient method for solving assignment problem
- based on Concept of opportunity cost.
- opportunity cost shows the relative penalties associated with assignment of resources to an activity as opposed to making the best or leat cost assignment
- We reduce the cost matrix to the extent of of having atleast one zeroin each row and each column,then it will be possible to make optimal assignment. (opportunity cost are all zero.)

METHOD OF SOLVING (MINMIZATION CASE)

- Step 1 :
- Determine the cost table from the given problem
  - if , number of sources= number of destinations ; go to step 3
  - if , number of sources not equal number of destinations ; go to step 2

- Step 2: make a square matrix, by adding dummy source or dummy destination.
- Step 3: subtract the smallest element of each row of the given cost matrix ,from each element of that row . (REDUCED MATRIX)
- Step 4 : subtract the smallest element of each COLUMN of the given cost matrix ,from each element of that COL. Each column and row ,have at least one zero.
- Step 5: in the modified matrix obtained ,search for optimal assignment.

How to search for optimal assignment

- Find a rows successively with single 0. en rectangle and cross off all other 0.continue ,until all rows have been taken care of.
- Repeat the procedure for each column of the reduced matrix.
- If a row or column has two or more zeroes one cant be chosen by inspection ,then assign arbitrary to any one of them and cross of all other 0.
- Repeat ,(a) to (c) ,until chain of assigning or cross ends .
- Step -6
- If the number of assignment is equal to the order of the cost matrix and optimum solution is reached.

If the number of assignment is less than the order of the cost matrix go to step8.

- Step -7
- Draw the min number of horizontal / vertical to cover of all the zeroes of the reduced matrix. How?

Procedure :

- ❖ mark (tick) rows that do not have any assigned zeroes .
- ❖ mark (tick) cols that have zeroes in the marked rows.
- ❖ mark (tick) rows that have any assigned zeroes in the marked cols.
- ❖ Repeat the steps ,until chain of marking is complete.
- ❖ Draw lines through ,all the unmarked rows and marked cols.this gives us desired minimum number of lines .

- STEP -8
- Develop the new revised cost matrix as :

a) Find the smallest element of the reduced matrix not covered by any of the lines.
b) Subtract the elements from all uncovered elements and add the same to all elements lying at the intersection of any two lines .

- STEP- 9
- Go to step 6 and repeat until an optimum sol is attained.

ILLUSTRATION

Example: A construction company has four large bulldozers located at four different garages. The bulldozers are to be moved to four different construction sites. The distances in miles between the bulldozers and the construction sites are given below.

How should the bulldozers be moved to the construction sites in order to minimize the total distance traveled?

Bulldozer \ Site	A	B	C	D
1	90	75	75	80
2	35	85	55	65
3	125	95	90	105
4	45	110	95	115

Step 1. Subtract 75 from Row 1, 35 from Row 2, 90 from Row 3, and 45 from Row 4.

$$\begin{bmatrix} 90 & 75 & 75 & 80 \\ 35 & 85 & 55 & 65 \\ 125 & 95 & 90 & 105 \\ 45 & 110 & 95 & 115 \end{bmatrix} \sim \begin{bmatrix} 15 & 0 & 0 & 5 \\ 0 & 50 & 20 & 30 \\ 35 & 5 & 0 & 15 \\ 0 & 65 & 50 & 70 \end{bmatrix}$$

STEP -2
Select 0 from col 1,0 from col 2,0 from col 3,5 from col 4

$$\begin{bmatrix} 15 & 0 & 0 & 5 \\ 0 & 50 & 20 & 30 \\ 35 & 5 & 0 & 15 \\ 0 & 65 & 50 & 70 \end{bmatrix} \sim \begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix}$$

Step 3. Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix}$$

- Step 4. Since the minimal number of lines is less than 4, we have to return to Step 5.

- 5 is the smallest entry not covered by any line,subtract 5 by each uncovered row

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix} \sim \begin{bmatrix} 15 & 0 & 0 & 0 \\ -5 & 45 & 15 & 20 \\ 30 & 0 & -5 & 5 \\ -5 & 60 & 45 & 60 \end{bmatrix}$$

- Now add 5 to each uncovered col.

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ -5 & 45 & 15 & 20 \\ 30 & 0 & -5 & 5 \\ -5 & 60 & 45 & 60 \end{bmatrix} \sim \begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix}$$

Now return to step 3
Cover all the zeroes of the matrix with the minimum number of horizontal lines /vertical lines

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix}$$

Step 4.:Since the minimal number of lines is less than 4, we have to return to Step 5.

Step 5 :

20 is the smallest entry not covered by line. Subtract 20 from each uncovered row.

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix} \sim \begin{bmatrix} 20 & 0 & 5 & 0 \\ -20 & 25 & 0 & 0 \\ 35 & 0 & 0 & 5 \\ -20 & 40 & 30 & 40 \end{bmatrix}$$

Then add 20 to each covered column.

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ -20 & 25 & 0 & 0 \\ 35 & 0 & 0 & 5 \\ -20 & 40 & 30 & 40 \end{bmatrix} \sim \begin{bmatrix} 40 & 0 & 5 & 0 \\ 0 & 25 & 0 & 0 \\ 55 & 0 & 0 & 5 \\ 0 & 40 & 30 & 40 \end{bmatrix}$$

- Now return to Step 3.
- Step 3. Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} \text{\sout{40}} & \text{\sout{0}} & \text{\sout{5}} & \text{\sout{0}} \\ \text{\sout{0}} & \text{\sout{25}} & \text{\sout{0}} & \text{\sout{0}} \\ \text{\sout{55}} & \text{\sout{0}} & \text{\sout{0}} & \text{\sout{5}} \\ \text{\sout{0}} & \text{\sout{40}} & \text{\sout{30}} & \text{\sout{40}} \end{bmatrix}$$

- Step 4. Since the minimal number of lines is 4, an optimal assignment of zeros is possible and we are finished .

$$\begin{bmatrix} 40 & 0 & 5 & \boxed{0} \\ 0 & 25 & \boxed{0} & 0 \\ 55 & \boxed{0} & 0 & 5 \\ \boxed{0} & 40 & 30 & 40 \end{bmatrix}$$

- Since the total cost for this assignment is 0, it must be an optimal assignment. Here is the same assignment made to the original cost matrix.

$$\begin{bmatrix} 40 & 0 & 5 & \boxed{0} \\ 0 & 25 & \boxed{0} & 0 \\ 55 & \boxed{0} & 0 & 5 \\ \boxed{0} & 40 & 30 & 40 \end{bmatrix}$$

- RESULT
- So we should send Bulldozer 1 to Site D, Bulldozer 2 to Site C, Bulldozer 3 to Site B, and Bulldozer 4 to Site A.

$$\begin{bmatrix} 90 & 75 & 75 & \boxed{80} \\ 35 & 85 & \boxed{55} & 65 \\ 125 & \boxed{95} & 90 & 105 \\ \boxed{45} & 110 & 95 & 115 \end{bmatrix}$$

THANK YOU!