Name of Course : **CBCS B.A. (Prog.)**

Unique Paper Code : **62351101**

Name of Paper : **Calculus**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find all the points of discontinuity of the functions f and g given by

$$f(x) = \begin{cases} 5x & ,\text{if } 0 \le x \le 1 \\ 6-x, & \text{if } 1 < x \le 2 \\ x^2 - 2x, & \text{if } 2 < x \le 3 \\ \dfrac{1}{x-3}, & \text{if } 3 < x \le 4 \\ 9x, & \text{if } 4 < x \le 5 \end{cases}$$

and

$$g(x) = (x - [x])^4 .$$

Also discuss the nature of discontinuity in each case.

2. Find the points at which the functions f and g are not differentiable, where

$$f(x) = |x+1| - |x-2| + 12|2x+5|$$

and

$$g(x) = \begin{cases} 5x \dfrac{e^{(x^2+1)/x} - e^{(x^2-1)/x}}{e^{(x^2+1)/x} + e^{(x^2-1)/x}}, & \text{if } x \ne 0 \\ 0, & \text{if } x = 0. \end{cases}$$

3. Find $y_n(0)$ if

(i). $y = \sin(6\sin^{-1} x)$

(ii). $y = 2\cos(\log x) + 3\sin(\log x)$.

If $z = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z \ .$$

4. Find the tangents and normal to the curve

$$x = 2a\cos\theta - a\cos 2\theta \, , y = 2a\sin\theta - a\sin 2\theta \quad \text{at} \quad \theta = \pi/2.$$

Find the radius of curvature at the origin for

$$2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0.$$

Trace the curve $y = x^3 - (x+1)^2$.

5. Write the Maclaurin's series expansion of $\log(2+3x)$. Verify Cauchy's Mean Value Theorem in each case:

(i). $f(x) = e^{5x}$, $g(x) = e^{-5x}$ in $[0,1]$

(ii). $f(x) = \sin 3x$, $g(x) = \cos 3x$ in $\left[-\frac{\pi}{6}, 0\right]$.

6. Evaluate the functions f and g for maximum and minimum values and separate the intervals of increasing and decreasing in each case, where f and g are given by

$$f(x) = 2x^5 - 10x^4 + 10x^3 - 15, \quad x \in \mathbb{R}$$

and

$$g(x) = 12x^5 - 45x^4 + 40x^3 + 36, \quad x \in \mathbb{R}\,.$$