Unique Paper Code:	32371501
Name of the Course:	B.SC (Hons.) STATISTICS: CBCS
Name of the Paper:	Stochastic Processes and Queuing Theory
Semester:	V
Duration:	3 hours
Maximum Marks:	75

Instructions for Candidates:

1. Attempt ***four*** questions in all.
2. All questions carry equal marks.
3. Use of calculator is allowed.

1. Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space $S = \{0,1,2,3,4,5,6\}$ and transition matrix

$$\begin{pmatrix} 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{5} & 0 & \frac{2}{5} & \frac{2}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Find the nature of each state. Also, identify the closed set(s), if any and obtain its stationary distribution (if it exists).

2. Let X be a zero- one truncated Poisson variate with parameter (λ) with zero-one classes missing and having the p.m.f,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!(1-e^{-\lambda} - \lambda e^{-\lambda})}, k = 2,3,.....$$

Obtain the probability generating function of r.v X and hence obtain mean and variance of X. Further, if X_1 and X_2 are i.i.d zero- one truncated Poisson variates with parameter (λ), find the probability generating function of $X_1 + X_2$.

3. Derive the differential difference equations for the linear growth process with immigration/emigration having

$\lambda_n = n\lambda + 4; \ \mu_n = n\mu - 3$

Hence obtain the second moment differential equation of mean population size when population starts with 3 individuals at t=0.

4. Under queuing model when arrival and departure follow Poisson process with single server , 5 as space limit for customers and FIFO queue discipline, derive the steady-state probabilities and obtain the mean size of the system . Further, modify above equations results when system has no space limit .

5. Construct the transition probability matrix when game starts with $8 and gambler with initial capital $3 starts the game and his probabilities of winning, losing and drawing a game are (0.46, 0.44,.10) respectively with stopping criteria is either gambler ruins or wins all amount. Further, find expected duration of the game when gambler either wins or loses a game with equal probability and no possibility of draw.

6. Assume that total number of COVID-19 cases follow Poisson process with parameter 6 per day. The COVID-19 patient recovery probability is 0.88 and probability of death is 0.12. Find the mean and variance of recovered cases at the end of 10^{th} day. Find, the probability that the number of recovered cases exceed number of deaths by 4 at the end of 2^{nd} day.