Unique Paper Code : 32371301

Name Of The Paper : Sampling Distributions

Name of the Course : B.Sc.(H) Statistics(under CBCS)

Semester : III

Duration : 3 hrs

M.Marks : 75

Instructions for candidates:

1. All questions carry equal marks
2. Attempt any four questions.

1) If X is a random variable and $E(X^2) < \infty$, then prove that $P(|X| \geq a) \leq E(X^2)/a^2$, $\forall a \geq 0$. If $\{X_k\}$ is a sequence of independent random variables such that $P(X_k = \pm k^{\alpha}) = \frac{1}{2}$, examine whether WLLN and central limit theorem hold for the sequence $\{X_k\}$.

2) Suppose $X_1, X_2, \ldots\ldots, X_n$ $(n > 1)$ are independent variates each distributed as $N(0, \sigma^2)$.

Find the p.d.f. of $W = \dfrac{X_1}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}}$ Why does W not follow the t-distribution?

If $V = \dfrac{W\sqrt{\sum_{i=1}^{n} x_i^2}}{\sqrt{\sum_{i=2}^{n} x_i^2}} \times \sqrt{\dfrac{n-1}{n}}$, obtain the distribution of V.

3) Show that for F- distribution with (n_1, n_2) d.f. , mean is greater than mode. Also discuss the shape of probability curve of F-distribution.

4) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n with common p.d.f.

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of r^{th} order statistic $X_{(r)}$. Also find COV $(X_{(1)}, X_{(n)})$.

5) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Derive the p.d.f. of S^2, where $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$. Also find $E(S^{2k})$. Hence or otherwise find $E(S^2)$and $Var(S^2)$.

6) The mean yield of two sets of plots and their variability are as given below Examine.

(i) Whether the difference in the mean yields of two sets of plots is significant and

(ii) Whether the difference in the variability in yields is significant.

	Set of 40 PLots	**Set of 60 Plots**
Mean Yield per plot	1258kg	1243kg
S.D. per Plot	34kg	28kg

Also obtain the 95% confidence interval for the difference of means.