

## SET-A

Unique paper code : 32371109

Name of the paper : Calculus

Name of the course : B.Sc.(Hons) Statistics (CBCS)

Semester : I

Duration : 3 Hours

Max. Marks : 75 Marks

### Instructions for candidates

Attempt four questions in all. All questions carry equal marks.

1. (i) Find stationary value of the function  $u = a^2 x^2 + b^2 y^2 + c^2 z^2$  subject to  $x^2 + y^2 + z^2 = 1$  and  $lx + my + nz = 0$ .

(ii) Solve the differential equations:

$$(D^3 + 2D^2 - 3D - 4)y = \cos x, \text{ where } D = \frac{d}{dx}.$$

2. (i) Evaluate  $\iint_{\left(\begin{array}{l} x>0, y>0 \\ \text{and} \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \end{array}\right)} \frac{\sqrt{1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)}}{\sqrt{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}}} dx dy.$

(ii) Solve the Lagrange's partial differential equation  $x(y-z)p + y(z-x)q = z(x-y)$ , where

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

3. (i) If  $y_1 = \frac{\sqrt{n}(x_1 - x_2)}{2\sqrt{x_1 x_2}}$  and  $y_2 = x_1 + x_2$  then show that  $\frac{\partial(y_1, y_2)}{\partial(x_1, x_2)} \times \frac{\partial(x_1, x_2)}{\partial(y_1, y_2)} = 1$ .

(ii) Show that  $\int_0^{\infty} \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{B(m, n)}{a^n(1+a)^m}.$

4. (i) Evaluate the limit:  $\lim_{n \rightarrow \infty} \frac{\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)^{1/2}\left(1 + \frac{3}{n}\right)^{1/3} \dots \left(1 + \frac{n}{n}\right)^{1/n}}{\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n}}.$

(ii) Show that differential equation  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$  is exact. Hence find its solution.

5. (i) If  $u = \sin^{-1} \left( \frac{x^{1/3} - 2y^{1/3}}{x^{1/2} - 3y^{1/2}} \right)^{1/2}$ , then show that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u (13 + \tan^2 u)}{144}$ .

(ii) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{e^x - e^{x \cos x}}{x - \sin x} \right)$ .

6. (i) If  $x^3 + y^3 - 4axy = 0$  then find the value of  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $(2a, 2a)$ .

(ii) Solve the partial differential equation  $z^2 = p q x y$ , where  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$ .



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