SET-A

Unique paper code : 32371109

Name of the paper : Calculus

Name of the course : B.Sc.(Hons) Statistics (CBCS)

Semester : I

Duration : 3 Hours

Max. Marks : 75 Marks

Instructions for candidates

Attempt four questions in all. All questions carry equal marks.

1. (i) Find stationary value of the function $u = a^2 x^2 + b^2 y^2 + c^2 z^2$ subject to $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$.

   (ii) Solve the differential equations:

$$\left(D^3 + 2D^2 - 3D - 4\right)y = \cos x, \text{ where } D = \frac{d}{dx}.$$

2. (i) Evaluate $\iint\limits_{\left(\substack{x>0,\ y>0 \\ \text{and} \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1}\right)} \frac{\sqrt{1-\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)}}{\sqrt{1+\frac{x^2}{a^2}+\frac{y^2}{b^2}}}\,dxdy$.

   (ii) Solve the Lagrange's partial differential equation $x(y-z)p + y(z-x)q = z(x-y)$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

3. (i) If $y_1 = \frac{\sqrt{n}\left(x_1 - x_2\right)}{2\sqrt{x_1 x_2}}$ and $y_2 = x_1 + x_2$ then show that $\frac{\partial(y_1, y_2)}{\partial(x_1, x_2)} \times \frac{\partial(x_1, x_2)}{\partial(y_1, y_2)} = 1$.

   (ii) Show that $\int_0^\infty \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}\,dx = \frac{B(m,n)}{a^n(1+a)^m}$.

4. (i) Evaluate the limit: $\lim\limits_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{1/2}\left(1+\frac{3}{n}\right)^{1/3}\ldots\left(1+\frac{n}{n}\right)^{1/n}}{\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \ldots + \frac{1}{8n}}$.

   (ii) Show that differential equation $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$ is exact.Hence find its solution.

5. (i) If $u = \sin^{-1}\left(\frac{x^{1/3} - 2y^{1/3}}{x^{1/2} - 3y^{1/2}}\right)^{1/2}$, then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u\left(13 + \tan^2 u\right)}{144}$.

   (ii) Evaluate $\lim_{x \to 0}\left(\frac{e^x - e^{x\cos x}}{x - \sin x}\right)$.

6. (i) If $x^3 + y^3 - 4axy = 0$ then find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(2a, 2a)$.

   (ii) Solve the partial differential equation $z^2 = p\,q\,x\,y$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.