SET-A (New Course)

Unique paper code : 32371109

Name of the paper : Calculus

Name of the course : B.Sc.(Hons) Statistics (CBCS)

Semester : I

Duration : 3 Hours

Max. Marks : 75 Marks

Instructions for candidates

Attempt four questions in all. All questions carry equal marks.

1 i) Prove ~~Euler,s~~Euler's theorem for the function

$z = (x + y)\,\Psi(\frac{y}{x})$, where Ψ is any arbitrary function.

ii) Use L'Hopital's rule to evaluate the following:

a) $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ (b) $\lim_{x\to 1}[\,(x-1)\tan(\frac{\pi x}{2})]$

2 i) Determine any local maxima or local minima of the function:

$f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x + 2y - 4z = 5$.

ii) If $l = x(1 - r^2)^{-1/2}$, $m = y(1 - r^2)^{-1/2}$, $n = z(1 - r^2)^{-1/2}$, where $r^2 = x^2 + y^2 + z^2$, then show that $J(l, m, n) = (1 - r^2)^{-5/2}$.

3 i) Form partial differential equation by the elimination of the constants a and b from

$(x - h)^2 + (y - k)^2 + z^2 = c^2$.

ii) Use Charpit's method to find the complete integral of $z^2(p^2z^2 + q^2) = 1$.

4 i) Solve $D^2 - 2D + 4y = e^x \cos x$.

ii) If $y = \sin(m\sin^{-1}x)$, then show that

$(1- x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$ and find $y_n(0)$.

5 i) Find the sum of the series sum as $n\to\infty$,

$\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \frac{n+3}{n^2+3^2} + \cdots \frac{1}{n}$.

ii) Prove Duplication formula. Use it to show that

$\beta(m, n) * \beta(m+\frac{1}{2}, m+\frac{1}{2}) = \pi m^{-1} 2^{1-4m}$.

6 i) Evaluate the following double integral:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{(a^2 - x^2 - y^2)}\ dydx.$$

Change the order of integration in the integral:

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{ax}} V dxdy.$$

ii) Solve:

$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.