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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 6712

Unique Paper Code : 32371109 HC

Name of the Paper : Calculus

Name of the Course : B.Sc. (Hons.) Statistics

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt *four* more questions by selecting *two* questions from each Section.

1. Attempt any *five* parts :

(a) If

$$f(x) = \begin{cases} x+2 & ; \quad x < 1 \\ 4x-1 & ; \quad 1 \le x \le 3 \\ x^2+5 & ; \quad x \ge 3 \end{cases}$$

Examine whether $\lim_{x \to 1} f(x)$ and $\lim_{x \to 3} f(x)$ exist.

P.T.O.

(b) Evaluate

$$\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-\sqrt{x^2+1}\right].$$

(c) Show that :

$$\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx.$$

(d) Evaluate the double integral :

$$\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)}\,\sqrt{(1-y^2)}}.$$

(e) Solve

$$x\frac{dy}{dx}+y=y^2\log x.$$

(f) Solve the differential equation :

$$(D^2+D+1)y=\sin 2x.$$

(g) Form a partial differential equation by elimination of th constants h and k from :

$$(x+h)^2+(y-k)^2+z^2=c^2.$$

(h) Solve :

$$y^2p-xyq=x(z-2y).$$

Section I

2. (a) A function $f(x)$ is defined as follows :

$$f(x)=\begin{cases}\dfrac{x^2}{a}-a & ; \quad x<a\\ 0 & ; \quad x=a\\ a-\dfrac{a^2}{x} & ; \quad x>a\end{cases}$$

$a\neq 0$. Show that $f(x)$ is continuous at $x = a$.

(b) Find the value of the nth derivative of the function

$y=e^{m\sin^{-1}x}$, for $x = 0$.

3. (a) If $\theta=t^n e^{-r^2/4t}$, find the value of n which will make :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}.$$

(b) Find the position and nature of the double points on the curve

$$y^3 = x^3 + ax^2.$$

4. (a) Show that :

$$\int_0^{\pi/2}\cos^{2m-1}\theta\sin^{2n-1}\theta\,d\theta=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$

(*b*) Evaluate

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}.$$

Section II

5. (*a*) Solve the differential equation :

$$\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2.$$

(*b*) Solve :

$$x^2y\,dx-(x^3+y^3)\,dy=0.$$

6. Solve any *two* of the following differential equations :

(*a*) $(D^3+1)y=\cos 2x$

(*b*) $\frac{d^2y}{dx^2}+4y=x\sin x$

(*c*) $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right).$

7. Solve the following partial differential equations :

(*a*) (*i*) $xz(z^2+xy)p-yz(z^2+xy)q=x^4.$

(*ii*) $(p^2+q^2)y=qz.$

(*b*) $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=2e^{2x}+3x^2y.$