[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5798 **H**

Unique Paper Code : 237152

Name of the Paper : Calculus – I

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Five** Questions in all.
3. Select **Three** Questions from **Section–I** & **Two** Questions from **Section–II.**

SECTION – I

1. (a) Find the nth derivative of $y = \sin^2 x \cos^3 x$.

(b) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, $xy \neq 0$, then

prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$. (7,8)

2. (a) Find the asymptotes of the curve

$$x^2y^2\left(x^2+y^2\right)^2 = \left(x^2+y^2\right)^2.$$

(b) Find the points of inflexion on the curve

$x = a(2\theta - \sin\theta)$, $y = a(2 - \cos\theta)$.

(c) If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$, then find the value of $\frac{\partial^2 y}{\partial x^2}$, when $t = \pi/2$. (5,5,5)

3. (a) If A and B are fixed points with co-ordinates (0,a) and (0,b) and P is a variable point (x,0) referred to rectangular axes then prove that $x^2 = a\,b$ when the angle APB is maximum.

(b) Trace the curve :

$$y^2x^2 = x^2 - a^2.$$ (5,10)

4. (a) Find the maximum or minimum values of x for the following equation :

$2x^3 - 21x^2 + 36x = 20$.

(b) If, $u = \log_e \sqrt{\left(x^2+y^2+z^2\right)}$, then show that

$$\left(x^2+y^2+z^2\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1.$$

(c) Find the position and nature of the double points on the following curve :

$ay^2 = (x - a)^2(x - b)^2$. (5,5,5)

SECTION – II

5. (a) Solve the initial value problem :

$(2x \cos y + 3x^2y)\,dx + (x^3 - x^2 \sin y - y)\,dy = 0$,

when, $y(0) = 2$.

(b) Solve :

$$y^2p^3 + 2.xp = y,\ p = \frac{dy}{dx}.$$

(c) Solve the following equation :

$p^2 + 2py \cot x = y^2$. (5,5,5)

6. (a) Solve the following equation :

$$x\left(y^2 - x^2 + a^2x\right)dx + y\left(y^2 + x^2 - b^2y\right)dy = 0.$$

(b) Solve the following equations :

(i) $x^2(x^2-1)\frac{dy}{dx}+x(x^2+1)y=(x^2-1)$,

(ii) $\frac{dy}{dx}=\frac{x-2y+5}{2x+y-1}$,

(iii) $x^2(x^2-1)\frac{dy}{dx}+x(x^2+1)y=(x^2-1)$. (6,9)

7. Solve the following differential equations :

(i) $\frac{d^2y}{dx^2}+2y=e^{2x}\cos 2x$

(ii) $(D^3+3D^2+2D)y=x^2$

(iii) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=\sin 3x$ (5,5,5)

(100)