[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1832** **GC-4**

Unique Paper Code : 32371202

Name of the Course : **B.Sc.(Hons.) Statistics**

Name of the Paper : Algebra

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **six** questions in **all**.

(c) Selecting **three** from each Section.

SECTION - A

1. (a) Form a cubic equation whose roots are the values of α, β, γ given by the relations $\sum\alpha = 3$, $\sum\alpha^2 = 5$, $\sum\alpha^3 = 11$. Hence find the value of $\sum\alpha^4$. 6

(b) If α, β, γ are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then form the equation whose roots are :

$$\frac{\alpha}{\beta+\gamma-\alpha}, \frac{\beta}{\gamma+\alpha-\beta}, \frac{\gamma}{\alpha+\beta-\gamma},$$

$6\frac{1}{2}$

2. (a) Solve the equation :
$3x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$,
given that the sum of two of its roots is equal to the sum of the other two.

$6\frac{1}{2}$

(b) Do the following vectors $a_1 = [1, 5, 7]$, $a_2 = [4, 0, 6]$, $a_3 = [1, 0, 0]$ form a basis for E^3? 3

(c) Given the basis vectors [1, 0, 0], [1,1,1], [0, 1,0] for E^3. Which vector can be removed from the basis and can be replaced by [4,3,3], while still maintaining a basis ? 3

3. (a) Define a circulant determinant. Show that

$$\begin{vmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{vmatrix}$$

has $a + b + c\lambda^2 + d\lambda^3$ as a factor where λ is a root of $x^4=1$. Hence show that the determinant is equal to $(a+b+c+d)(a-b+c-d)\{(a-c)^2+(b-d)^2\}$. 6

(b) Express

$$\begin{vmatrix} (1+ax)^2 & (1+ay)^2 & (1+az)^2 \\ (1+bx)^2 & (1+by)^2 & (1+bz)^2 \\ (1+cx)^2 & (1+cy)^2 & (1+cz)^2 \end{vmatrix}$$

as a product of two determinants and hence evaluate it. $6\frac{1}{2}$

4\. (a) Solve the following system of equations with the help of Cramer's rule : $6\frac{1}{2}$

$ax + by + cz = k,$

$a^2x + b^2y + c^2z = k^2,$

$a^3x + b^3y + c^3z = k^3.$

(b) When is a matrix said to be in : 4

(i) Echelon form,

(ii) Reduced echelon form ?

(c) Find the area of the parallelogram whose vertices are 2

(–2, –2), (0,3), (4,–1), (6, 4).

SECTION - B

5\. (a) If B and C are square matrices of order n and if A=B + C, then show that :

$A^{P+1} = B^P [B + (p + 1) C]$, provided B and C commute, $C^2 = \mathbf{0}$ and p is a positive integer. 6

(b) Define Orthogonal and Unitary matrices.

If A is a square matrix, $A-\frac{1}{2}I$ and $A+\frac{1}{2}I$ are orthogonal (I is an identity matrix of order same as A), then prove that A is skew symmetric and $A^2=-\frac{3}{4}I$. Also deduce that A is of even order. $6\frac{1}{2}$

6\. (a) Define elementary matrices. Show that elementary matrices are non-singular. Also obtain their inverses. 6

(b) Find the characteristic roots of the matrix

$$A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & -3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

and show that the characteristic vectors associated with its distinct characteristic roots are mutually orthogonal.

$6\frac{1}{2}$

7. (a) State Cayley-Hamilton theorem. Given $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, express $A^4-4A^3-A^2+2A-5I$ as a linear polynomial in A and hence evaluate it. 6

(b) Reduce the real quadratic form $3x^2-3y^2-5z^2-2xy-6yz-6zx$ to its canonical form and find its rank, signature and index. $6\frac{1}{2}$

8. (a) If G is a generalized inverse of X'X, then prove that :

(i) G' is also a generalized inverse of X'X,

(ii) XGX'X=X,

(iii) XGX' is invariant of G,

(iv) XGX' is symmetric whether G is symmetric or not. 6

(b) Derive the formula to find the inverse of a non-singular matrix M of order nxn, partitioned as :

$$M = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$

Where α, β, γ and δ are the block matrices of order sxs, sxm, mxs and mxm respectively, and α is a non-singular matrix. $6\frac{1}{2}$