[This question paper contains 5 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5799 **H**

Unique Paper Code : 237153

Name of the Paper : Algebra – I

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from each section.

SECTION I

1. (a) If α, β, γ are roots of the equation $x^3 - x^2 + x - 1 = 0$, then find the value of

(i) $\sum(\alpha - 9)$

(ii) $\sum(\alpha\beta - 1)$

(iii) $\sum\alpha(1 - \beta\gamma)$

(b) Diminish the roots of the equation

$x^4 - 16x^3 - 8x^2 + 4x + 1 = 0$ by 2.

(c) Find the equation whose roots are the square of the roots of equation $x^3 - 10x^2 + 9x - 1 = 0$. (4½,4,4)

2. (a) Find the modulus and argument of complex number $\frac{(\sin\alpha + i\cos\alpha)^4}{(\cos\alpha + i\sin\alpha)^3}$.

(b) If $z = \cos\theta + i\sin\theta$ then prove that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$.

(c) Find all the values of $(1+i\sqrt{3})^{\frac{z}{3}}$. (4½,4,4)

3. (a) Show that, for any positive integer $n > 1$, $n! < \left(\frac{n+1}{2}\right)^n$.

(b) If a, b, x and y are real numbers such that $a^2 + b^2 = x^2 + y^2 = 1$ then prove that $ax + by \le 1$.

(c) If a, b, c represent length of sides of a triangle taken in order, then prove that

$9(a^3 + b^3 + c^3) > (a + b + c)^3$. (4½,4,4)

4. (a) Solve, $x^3 - 7x^2 + 14x - 8 = 0$, given that the roots of equation are in GP.

(b) If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$, then show that $\sum \sin 2\alpha = \sum \cos 2\alpha = 0$.

(c) If a, b and c are three positive numbers then, show that

$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \ge 3$. (4½,4,4)

SECTION II

5. (a) If $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, then prove that

$(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the two rowed identity matrix, n is a positive integer and a and b are arbitrary scalars.

(b) Prove that

(i) $tr(AA') \ge 0$

(ii) $tr(A^2) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2$, if $A = (a_{ij})$ is a symmetric matrix of order n.

(c) Define Idempotent and Nilpotent matrices. (4½,5,3)

6. (a) Prove that

$$\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$$

(b) Let e be the column vector with elements (1,1,1,...1) and e' its transposed row vector. Let A be n- square matrix and I the identity matrix. Let the matrix M(x) be given by $M(x) = I + xAe\,e'$ where x is a scalar.

(i) Prove that $M(x)\,M(y) = M(x + y + kxy)$, where k is the scalar $e'Ae$.

(ii) Verify that reciprocal of M(x) is $M\left(\frac{-x}{1+kx}\right)$.

(iii) Show that the matrix $R = (r_{ij})$ where $r_{ii} = 1$; $r_{ij} = \rho$, $i \neq j$ can be written as $(1-\rho)I + \rho e e'$. Hence find the reciprocal of this matrix. (5,7½)

7. (a) Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a Skew–Hermitian matrix.

(b) Use determinants to solve the following equations :

$$ax + by + cz = 1$$

$$a^3x + b^2y + c^2z = k$$

$$a^3x + b^3y + c^3z = k^2$$

(7,5½)

8. (a) If A is a non singular matrix of order n, then show that

(i) $|adj A| = |A|^{n-1}$

(ii) $adj(adj A) = |A|^{n-2}. A$

(iii) $|adj(adj A)| = |A|^{(n-1)^2}$

(b) Show that the possible square roots of the two rowed identity matrix I are

$\pm I$ and $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ where $1 - \alpha 2 = \beta\gamma$. (7½,5)