Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351502**
Name of Paper	: **C 12-Group Theory-II**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. If Aut (G) and Inn (G) denote the set of all automorphisms and the set of all inner automorphisms, respectively of a group G, then obtain Aut $(\mathbb{Z}_8)$ and Inn (D_8), where D_8 is a dihedral group of order 8. Is Inn (D_8) isomorphic to $\mathbb{Z}_4$? Justify.

2. Express $U(40)$ as an External Direct Product and an Internal Direct Product of its subgroups. What are the possible order of elements in $U(40)$? How many elements are of order 4 in $U(40)$? Further, find the number of cyclic subgroups of order 4 in $U(40)$.

3. Determine the isomorphism classes of Abelian groups of order 500. Further find those isomorphism classes that have 'only one subgroup of order 5' and those isomorphism classes that has 'only six subgroups of order 5'.

4. Let $G = D_8$ be the dihedral group of order 8 with the usual generators r and s and $A = \langle sr \rangle$ be the subgroup in G. Find the centralizer of A, $C_G(A)$ and the normalizer of A, $N_G(A)$. Further, list the left costs of H, and label them with the integers 1,2,3,4. Exhibit the image of each element of G under the representation π_H of G into S_4 obtained from the action of G by left multiplication on the set of left cosets of H in G. Is the representation faithful? Justify.

5. Let $\sigma_1 = (25)(136)$ and $\sigma_2 = \sigma_1(65)$ be the permutations in S_7. Are σ_1 and σ_2 conjugate? If they are, give an explicit permutation τ such that $\tau\sigma_1\tau^{-1} = \sigma_2$. Also, discuss again for $\sigma_2 = \sigma_1^5$. Further find $|C_{S_7}(\sigma)|$, where $\sigma = (25)(136)$.

6. Let G be a group of order 735. If the number of Sylow 7-subgroups are more than 1, then show that there exists a normal Sylow 5-subgroup, K. Also show that there exists a normal subgroup of order 245. Further show that $K \subseteq Z(G)$.