Name of the Course : **B.Sc. (H) Mathematics**

Unique Paper Code : **32357505**

Name of the Paper : **DSE-II Discrete Mathematics**

Semester : **V Semester**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Show that the set $A = \{2,\ 3, 4, 6, 8, 24, 48\}$ with divisibility as the relation is a partial ordered set. Draw the Hasse diagram of A. Is it a chain? Justify your answer. Find out the maximal and minimal elements of A. Also, determine the least and greatest elements of A. Give subset of A which is a chain and a subset of A which is an antichain with regard to the same partial order relation.

Consider the partial ordered sets $(L, \subseteq)$ where $L = P(S)$ is the power set of a non-empty set S and $(Q, \leq)$ where $Q = \{0,1\}$ and $0 < 1$.

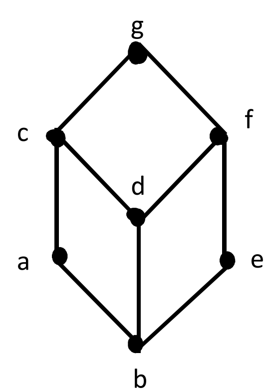
Consider $\varphi : L \to Q$ defined by

$$\varphi(U) = \begin{cases} 1, & \text{if } U = S \\ 0, & \text{if } U \neq S \end{cases}.$$

Is φ order preserving? Justify your answer.

2. Consider lattices $L_1 = \{\ 1, 5\}$ and $L_2 = \{\ 1, 3, 9\}$ with divisibility as the partial order relation. Is $L_1 \times\ L_2$ a lattice? If yes, then state the partial order relation on $L_1 \times\ L_2$ and draw its Hasse diagram. Prove that a finite lattice always has the greatest element and least element.

Consider lattice L_3 represented by the Hasse diagram shown below



Find out 5 sublattices of L_3. Is union of any two sublattices of L_3 a sublattice of L_3? Justify your answer by providing examples.

3. Prove or disprove the following two statements

i) sublattice of a modular lattice is modular

ii) sublattice of a distributive lattice is distributive.

Verify whether or not the lattice $L = (\{1, 2, 3, 6, 10, 12, 24, 60, 120\}$, GCD, LCM) is modular.

Find the disjunctive normal form of the given polynomial $p = x(y+z)' + (xy + z')x$.

Also, find the conjunctive normal form of above polynomial p.

4. Does the expression $x'z'$ imply the expression $xy'z' + x'y + x'y'z' + x'yz$. Give reasons for your answer.

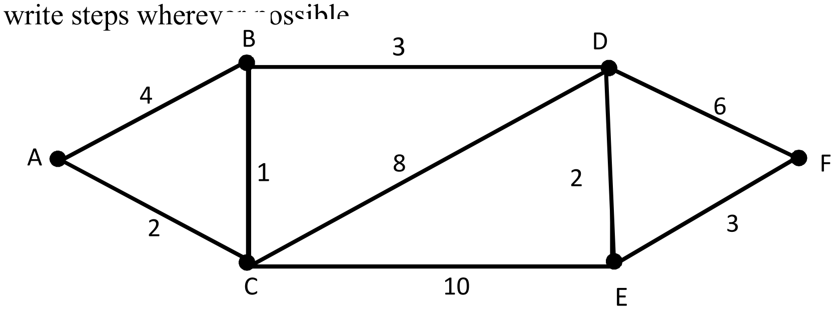
Find the prime implicants of $p = xyz + xyz' + x'y\,z' + x'y'z$.

Also reduce the above polynomial p into minimal sum of products form using Quine−McCluskey method or Karnaugh maps.

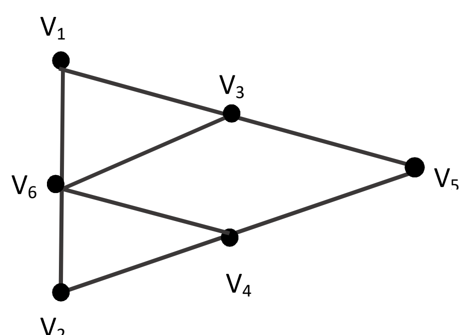
Draw the contact diagram of the circuit $q = (xy')' + (x' + y + z)' + xz$.

Further, give the symbolic representation of the above circuit q.

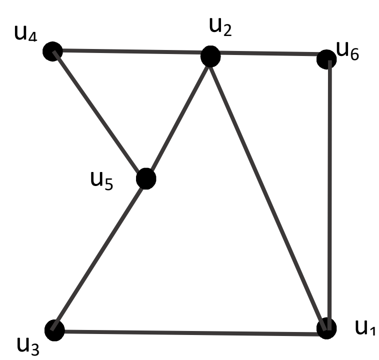
5. Apply Dijkstra's Algorithm OR improved version of Dijkstra's Algorithm to find a shortest path from A to F, also write steps wherever possible.



Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below.



G_1



G_2

Find a permutation matrix P such that $A_2 = PA_1P^{\mathrm{T}}$.

6. Explain the Königsberg bridge problem and discuss the solution provided by graph theory to this problem. In the graph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists. Is the graph Hamiltonian? Display a Hamiltonian cycle or explain clearly why no such cycle exists.