

Name of Course	: CBCS B.A. (Prog.)
Unique Paper Code	: 62351101
Name of Paper	: Calculus
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

*Attempt any four questions. All questions carry equal marks.*

1. Examine the continuity of the following functions

(i).  $f: (0, \infty) \rightarrow \mathbb{R}$  defined as

$$f(x) = \begin{cases} 7x, & \text{if } 0 < x \leq 1 \\ 2 - x, & \text{if } 1 < x \leq 2 \\ x^2 - 2x, & \text{if } 2 < x \leq 4 \\ x + 4, & \text{if } x > 4 \end{cases}$$

at  $x = 1, 2$  and  $4$ .

(ii).  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined as

$$f(x) = \begin{cases} \frac{5xe^{5/x}}{1 + e^{5/x}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

at  $x = 0$ .

(iii).  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined as

$$f(x) = \begin{cases} -2x^3, & \text{if } x \leq 0 \\ 5x - 8, & \text{if } 0 < x \leq 1 \\ x^2 - 3x, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$$

at  $x = 0, 1$  and  $x = 2$ .

2. If  $y = [2x + 2\sqrt{1+x^2}]^m$ , then prove that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

Also, verify Euler's Theorem for

$$z = 2^n x^n \log \frac{y}{4x}.$$

3. Find the asymptotes of the following curves

(i).  $(x^2 - y^2)^2 - 4x^2 + x = 0$

(ii).  $x^2y + xy^2 + xy + y^2 + 3x = 0$ .

Also, trace the curve given by

$$y^2(2+x) = 6x^2 - x^3.$$

4. Verify Lagrange's Mean Value Theorem for the function given by

$$f(x) = (2x - 1)(x - 3)(2x - 5) \text{ for } x \in [0,4]$$

and apply it to prove that

$$\sqrt{1+x} < 1 + \frac{1}{2}x \quad \text{if } x \in (-1, \infty), x \neq 0.$$

Also, verify Cauchy's Mean Value Theorem for following functions

$$f(x) = x^2 - 3x - 5, \quad g(x) = x^2 + 2x - 1 \text{ in } [0,1].$$

5. Find the equation of the tangent and normal at the point ' $\theta$ ' to the curve

$$x = 6 \cos^3 \theta, \quad y = 6 \sin^3 \theta.$$

Find the radius of curvature at the origin for the following curves:

(i).  $x^4 - 4x^3 - 18x^2 - y = 0.$

(ii).  $x = 5(\theta + \sin \theta), \quad y = 5(1 - \cos \theta).$

6. Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^2 \sin x^2} \quad \text{and} \quad \lim_{x \rightarrow 0^+} (\cot x)^{\sin x}.$$

Find the maximum and minimum values of the function  $f(x) = 3x^5 - 15x^4 + 15x^3 - 1$  and separate the intervals in which the function  $g(x) = 2x^3 - 9x^2 + 12x - 5$  is increasing or decreasing.