

Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32351102
Name of Paper	: BMATH102-Algebra
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

- Find all the rational roots of the equation $224y^3 - 344y^2 + 22y - 15 = 0$ and also solve the equation $16y^4 - 96y^3 + 56y^2 + 264y - 135 = 0$ given that the roots form an arithmetical progression.
- Draw a rough sketch of the region corresponding to the inequality $\frac{1}{\sqrt{2}} < |z - 1 - i| < \sqrt{2}$. Use De Moivre's theorem to find the square root of $-3 + 4i$. Find the extended argument $Arg z$ of the complex number $z = (-\sqrt{3} - i)(1 + i)$.
- Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define a relation R_1 on the set A which is an equivalence relation. Define a relation R_2 on the set A which is not an equivalence relation. Let a be an integer, prove that there exists an integer k such that $a^2 = 5k$ or $a^2 = 5k + 1$. Evaluate $(5.6 + 8.11 + 19.23) \pmod{9}$.
- Show that the function $f: \left(\frac{2}{5}, \infty\right) \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \left(\frac{2}{5}, \infty\right)$ defined by $f(x) = \log_5(5x - 2)$ and $g(x) = \frac{5^x + 2}{5}$ are the inverse of each other. Prove that the interval $(3, 7)$ and $(1, \infty)$ have the same cardinality. Show that 314 and 159 are relatively prime integers.
- Describe the solutions of the following system in parametric vector form. Give a geometrical description of the solution set.

$$\begin{aligned} 4x_1 - 2x_2 + 6x_3 &= 8 \\ x_1 + x_2 - 3x_3 &= -1 \\ 15x_1 - 3x_2 + 9x_3 &= 21 \end{aligned}$$

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation which first reflects points through the line $x_1 = x_2$ and then rotates points (about the origin) through $\pi/4$ radians. Find the standard matrix of T .

- Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$. Find a basis for
 - Row Space of A .
 - Null Space of A .

Also find Rank A and Nullity A .