

Name of Course	: B.A. (Prog.) Mathematics
Unique Paper Code	: 62354343
Name of Paper	: Analytical Geometry and Applied Algebra
Semester	: III
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Describe, sketch and label the focus, vertex and directrix of the parabola

$$y^2 - 2y - 12x - 23 = 0$$

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$4x^2 + y^2 + 16x - 2y + 13 = 0$$

and sketch it.

2. Describe, sketch and label the centre, vertices, foci and asymptotes of the hyperbola

$$9x^2 - 4y^2 - 18x + 45 = 0$$

Find the equation of parabola having vertex $(1,2)$ and focus $(-3,2)$. What is its directrix?

Find the equation of ellipse having length of the major axis 26 and foci $(\pm 12, 0)$.

3. Describe $x^2 + y^2 + z^2 - 2x + 6y - 2z + 12 = 0$.

Find a vector \mathbf{v} of length 5 along the direction of $-i + 6j + k$ and also find the angle between \mathbf{v} and $2i + 6j + 2k$.

Find the direction cosines of $\mathbf{v} = 2i + 2j - 2k$. Use the direction cosines to approximate the direction angles of \mathbf{v} . Also find the vector component of \mathbf{v} orthogonal to $\mathbf{b} = i + j - k$.

4. Consider the parallelepiped with adjacent edges $\mathbf{u} = j + k$, $\mathbf{v} = i + 2k$, $\mathbf{w} = i + 3j + 3k$. Find the area of the face determined by \mathbf{u} and \mathbf{w} and find the angle between \mathbf{u} and the plane containing the face determined by \mathbf{v} and \mathbf{w} .

Find the distance from the point $P(-2, 2, 3)$ to the line through $A(2, 2, 0)$ and $B(-1, 4, -3)$.

Find the equation of the sphere that has center in the first octant and is tangent to each of the three coordinate planes.

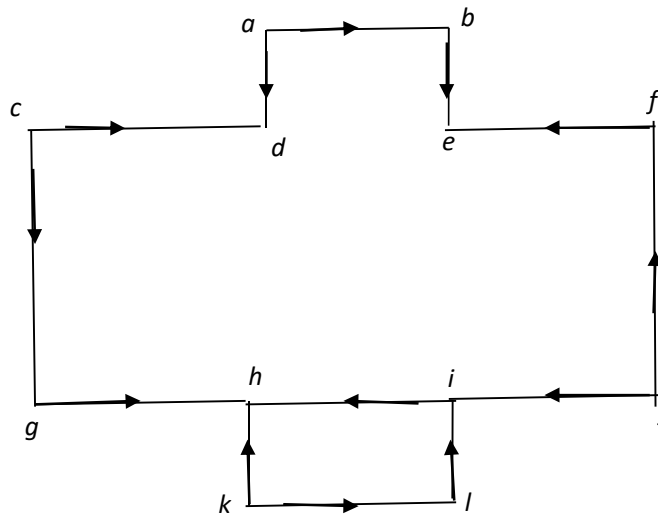
5. Find the distance between the skew lines:

$$L_1: x = 3 + 8t, y = 6 - 8t, z = 10t \quad L_2: x = 3 + 8t, y = 5 - 3t, z = 6 + t$$

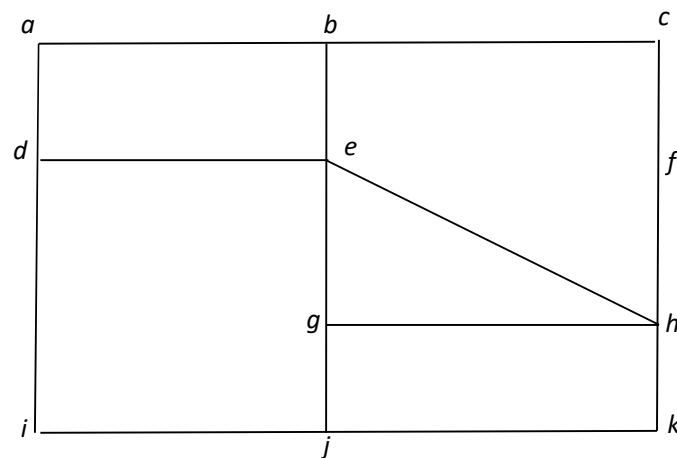
Determine whether the points $P_1(-6, 4, 8)$, $P_2(9, -2, 0)$ and $P_3(1, -5, 3)$ lie on the same line.

Find the equation of the plane through the points $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$.

6. Find a vertex basis for the following graph. Justify your answer.



Find a minimal edge cover for the following graph. Give a detailed logical analysis.



Three pitchers of sizes 8 litres, 5 litres and 3 litres are given. If initially 8 litres pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly one litre of water in one pitcher.