Question Bank

Numerical Analysis

Q1: How many roots of the function $f(x) = (x^2-1)(x-3)$ lie in the interval $[-4, 4]$ and how many can be calculated using the Bisection method? Also find the root/roots using Bisection method.

Q2: While computing the root of $f(x) = \cos x - x$ on the interval, to acheive the accuracy within 6 significant digits, how many iteration do we need to perform?

Q3: Can we calculate the root of function $f(x) = X^2 - 2x + 1$ on interval $[-3, 6]$ using Bisection method? Justify.

Q4: Perform 4 iteration of Bisection method to find out the root of function $g(x) = 2x\cos(\pi x) - e^{x-1} = 0$.

Q5: Perform 4 iteration of Regula Falsi method to find out the root of function $g(x) = 2x\cos(\pi x) - e^{x-1} = 0$.

Q6: Derive the formula for Newton Raphson Method. Solve the equation $x^2 + 4x\breve{}9 = 0$ using Newton Raphson method.Discuss drawbacks of the Newton Raphson method.

Q7: An iteration method ois defined by

$$x_{n+1} = \frac{2(x_n^3 + 1)}{1 + 3x_n^2}, \quad n = 0, 1, 2, \ldots$$

Find the positive real quantity to which the method converges. Hence, determine the rate of convergence of the method.

Q8:Solve the following system of equations using Gauss Elimination method with partial pivoting:

$$\begin{aligned} 2x + 2y - z &= 6 \\ 4x + 2y + 3z &= 4 \\ x + y + z &= 0 \end{aligned}$$

Q9: Perform 5 iterations of Regula Fasli and Secant Method to calculate the root of function $x^2 - 2 = 0$ on the interval $[0, 2]$. Which method will converge faster to the exact root? Justify the order of convergence numerically.

Q10:Solve the following system of equations using Gauss Elimination method with scaled partial pivoting:

$$\begin{aligned} 2x + 2y - z &= 6 \\ 4x + 2y + 3z &= 4 \\ x + y + z &= 0 \end{aligned}$$

Q11: Use Gaussian elimination with partial Pivoting and scaled partial pivoting with 4-digit rounding arithmetic to find the solution of following system of linear equations.

$$\begin{aligned} 3.03x + 12.1y + 14z &= -119 \\ -3.03x + 12.1y - 7z &= 120 \\ 6.11x - 14.2y + 21z &= -139 \end{aligned}$$

Compare the approximate solution with the exact solution $\overline{x} = (0, 10, 17)^T$.

Q12:Solve the following system of equations using LU decomposition:

$$\begin{aligned} 2x + 2y - z &= 6 \\ 4x + 2y + 3z &= 4 \\ x + y + z &= 0 \end{aligned}$$

Q13: For any square matrix A of order n, if the LU decomposition exists, it is not unique. Justify.

Q14: Perform 4 iterations of Gauss Jacobi method to solve the following system of equation with initial approximation $\overline{x}^0 = (0, 0, 0)^T$:

$$\begin{aligned} 28x + 4y - z &= 32 \\ x + 3y + 10z &= 24 \\ 2x + 17y + 4z &= 35 \end{aligned}$$

Q15: Perform 4 iterations of Gauss Seidel method to solve the following system of equation with initial approximation $\overline{x}^0 = (0, 0, 0)^T$:

$$\begin{aligned} 28x + 4y - z &= 32 \\ x + 3y + 10z &= 24 \\ 2x + 17y + 4z &= 35 \end{aligned}$$

Q16: Perform 4 iterations of SOR method with $\omega = 1/3$ to solve the following system of equation with initial approximation $\overline{x}^0 =$

$(0, 0, 0)^T$:

$$\begin{aligned} 28x + 4y - z &= 32 \\ x + 3y + 10z &= 24 \\ 2x + 17y + 4z &= 35 \end{aligned}$$

Q17: Consider the following system of equation

$$\begin{aligned} 28x + 4y - z &= 32 \\ x + 3y + 10z &= 24 \\ 2x + 17y + 4z &= 35 \end{aligned}$$

Will the sequence of approximations $\overline{x}^{(n)}$ obtained using Gauss Jacobi and Gauss Seidel method converge for any initial approximation $\overline{x}^0$?

Q18 :Let $P(x)$ be the interpolating polynomial for the data $(1, 4), (3, 2), (5, y),$ and $(8, 5)$. The coefficient of x^3 in $P(x)$ is $180/31$. Find y.

Q19 : Prove the following relations:

1. $(1 + \Delta)(1 - \nabla) \equiv 1$
2. $\mu\delta \equiv \dfrac{\Delta + \nabla}{2}$
3. $\Delta\nabla \equiv \nabla\Delta \equiv \delta^2$
4. $E^{1/2} =\equiv \mu + \dfrac{\delta}{2}$
5. $\nabla - \Delta = -\Delta\nabla$
6. $1 + \delta^2\mu^2 \equiv \left(1 + \dfrac{\delta^2}{2}\right)^2$

Q20 : A fourth degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24; \Delta^3 P(0) = 6;$ and $\Delta^2 P(0) = 0;$ where $\Delta P(x) = P(x + 1) - P(x)$. Compute $\Delta^3 P(3)$.

Q21 : Using Lagrange Interpolation formula find the polynomial that approximates the following data:

x	0	1	2	3
$f(x)$	1	2	1	10

Q22 : Create a divided difference table for the following table: Us-

x	4	5	7	10	11	18
$f(x)$	48	100	294	900	1210	2028

ing Newton's divided difference formula, find the value of $f(8)$ and $f(13)$.

Q23 : Verify that the following difference approximations for calculating first order derivative $f^{'(x)}$ for any function $f(x)$ are

1. second order accurate

$$f^{'}(x) = \frac{f(x+h) - f(x-h)}{2h}$$

2. second order accurate

$$f^{'}(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

3. second order accurate

$$f^{'}(x) = \frac{3f(x) - f(x-h) + f(x-2h)}{2h}$$

4. fourth order accurate

$$f^{'}(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

Q24 : Verify that the following difference approximations for calculating second order derivative $f^{''}(x)$ for any function $f(x)$ are

1. second order accurate

$$f^{''}(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{2h}$$

2. second order accurate

$$f^{''}(x) = \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^3}$$

3. second order accurate

$$f^{''}(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^3}$$

4. fourth order accurate

$$f^{''}(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x)168f(x-h) - f(x-2h)}{12h^2}$$

Q25 : Let $f(x) = x^2 + e^x - 2x$ and $x = 0$. Show numerically that the approximation of derivative of $f(x)$ at $x = 0$ is

1. second order accurate using the difference formula
$$f^{'}(x) = \frac{f(x+h) - f(x-h)}{2h}$$
2. second order accurate using the difference formula
$$f^{'}(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$
3. second order accurate using the difference formula
$$f^{'}(x) = \frac{3f(x) - f(x-h) + f(x-2h)}{2h}$$
4. fourth order accurate using the difference formula
$$f^{'}(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

Q26 : Let $f(x) = x^2 + e^x - 2x$ and $x = 0$. Show numerically that the approximation of second order derivative of $f(x)$ at $x = 0$ is

1. second order accurate using the difference formula
$$f^{''}(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{2h}$$
2. second order accurate using the difference formula
$$f^{''}(x) = \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^3}$$
3. second order accurate using the difference formula
$$f^{''}(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^3}$$
4. fourth order accurate using the difference formula
$$f^{''}(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x)168f(x-h) - f(x-2h)}{12h^2}$$

Q27 : Evaluate
$$\int_1^2 \frac{dx}{1+x^3}$$
using Trapezoidal Rule, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule.

Q28 : The velocity of an object as a function of time is given by:

Time(t)	2	5	7	9	12
velocity($v(t)$)	12	16	24	15	33

Using the trapezoidal rule and Simpson's $\frac{1}{3}$ rule, calculate the distance travelled by the object between $t = 2$ and $t = 12$.

Q29 : Using Euler's method, Modified Euler's Method and fourth order Runge-Kutta method, approximate the solution of the given over indicated time interval and number of time steps:

1. $x^{'}(t) = tx^3 - x, \quad (0 \leq t \leq 1), \quad x(0) = 1, \quad N = 4.$
2. $x^{'}(t) + 4x/t = t^4, \quad (0 \leq t \leq 3), \quad x(0) = 1, \quad N = 5.$
3. $x^{'}(t) = \dfrac{\sin x - e^t}{\cos x}, \quad (0 \leq t \leq 1), \quad x(0) = 0, \quad N = 3.$
4. $x^{'}(t) = \dfrac{1 + x^2}{t}, \quad (1 \leq t \leq 4), \quad x(1) = 0, \quad N = 5.$
5. $x^{'}(t) = t^2 - 2x^2 - 1, \quad (0 \leq t \leq 1), \quad x(0) = 0, \quad N = 6.$
6. $x^{'}(t) = x + t, \quad (1 \leq t \leq), \quad x(1) = 0, \quad N = 10.$
7. $x^{'}(t) = x + t + xt, \quad (0 \leq t \leq 0.1), \quad x(0) = 1, \quad N = 10.$
8. $x^{'}(t) = \dfrac{t}{x}, \quad (0 \leq t \leq 1), \quad x(0) = 1, \quad N = 5.$

Also calculate the order of convergence numerically.