Question Bank

B.Sc(H) Mathematics-VI Semester

Complex Analysis

Q 1. If z_1 and z_2 are two complex numbers prove that $\left|\frac{z_1 - z_2}{1 - z_2\overline{z_1}}\right| = 1$ if either $|z_1|$=1 or $|z_2|$=1.

What exception must be made if $|z_1|$=1 and $|z_2|$=1.

Q 2. If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$. Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$.

Q 3. Show that if $f(z)$ is a differentiable function, then the CR equation can be put in the form

$\frac{\partial f}{\partial z} = 0.$

Q 4. Find the analytic function $f(z) = u + iv$, if $u + v = \frac{\sin 2x}{\cos h\ 2y - \cos\ 2x}$.

Q 5. Given $v(x, y) = x^4 - 6x^2x^2 + y^4$, find the $f(z) = u(x, y) + iv(x, y)$ such that $f(z)$ is analytic.

Q 6. Find the analytic function $f(z) = u + iv$, if $u - v = e^x(\cos y - siny)$.

Q 7. If $f(z)$ is analytic function prove that $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|f(z)|^2$=4$|f'(z)|^2$.

Q 8. Find the real part of the analytic function whose imaginary part is

$e^{-x}[2xy \cos y + (y^2 - x^2) \sin y]$. Construct the analytic function.

Q 9. Find the image of the strip $2 < x < 3$ under map $w = \frac{1}{z}$.

Q 10. Show that the bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

Q 11. Find the anti-derivative of the function $(z) = iz + z^2 + 2e^{-iz}$. Also, use the ML-inequality to prove that

$$\left|\int \frac{f(z)}{z} dz\right| \leq 4(1 + e)\pi$$

on the positively oriented circle C: |z| = 1.

Q 12. Let C be the positively oriented circle $|z - i| = 3$. Use the Cauchy Integral Formula to evaluate on C

$$\left|\int \frac{e^{i\pi z}}{(z-1)(z-2)}\,dz\right|.$$

Use the extension of Cauchy Integral Formula to find the value of the integral

$$\int \frac{e^{i\pi z}}{(z-1)^4}\,dz.$$

What is the value of the integral

$$\int \frac{e^{i\pi z}}{(z-5)}\,dz.$$

Justify your answer.