Question Bank

Bio-Mathematics

Q1. When the drug theophylline is administered for asthma, a concentration below $3mg\ l^{-1}$ has little effect and undesirable side-effects appear if the concentration exceeds $12mg\ l^{-1}$. For a body that weighs W kg, the concentration when M mg is present is $4M/Wmg\ l^{-1}$. If the constant that measures the rapidity at which the concentration falls is $T = 5h$, Find the concentration at time t h after an initial dose of D mg. If D = 600 and W = 70, when is a second dose necessary to prevent the concentration from becoming ineffective. What is the shortest safe time interval t_0 at which doses of $600mg$ can be given regularly?

Q2. What is a limit cycle? Verify the existence/nonexistence of the limit cycle in the following dynamical system:

$$\dot{x} = x + y - x(x^2 + 2y^2)$$
$$\dot{x} = -x + y - y(x^2 + 2y^2).$$

Q3. For the following system find the fixed points, classify them, sketch the neighboring trajectories and try to fill the rest of the phase portrait:

$$\dot{x} = xy - 1$$
$$\dot{x} = x - y^3.$$

Q4. Plot the phase portrait and classify the fixed points of the linear system

$$\begin{aligned}\dot{x} &= -3x + 2y \\ \dot{y} &= x - 2y\end{aligned}$$

Also find the solution.

Q5. Explain the working of the heartbeat cycle. What are three basic features on which heartbeat model is developed? Develop the basic heartbeat model that reflects the rapid return to the equilibrium state.

Q6. Describe the mathematical model of population growth for any species changing by birth only. In a research on population dynamics of mosquitoes, it was estimated that the initial population is 2000. Over the time period of one month, 300 births and 100 deaths were recorded in the population. Predict the population size at the end of 10 months.

Q7. State Law of Mass Action and derive the mathematical model governed by intermediaries X and Y in trimolecular reaction:

$$
\begin{aligned}
&A \longrightarrow X \\
&B + X \longrightarrow Y + D \\
&2X + Y \longrightarrow 3X \\
&X \longrightarrow E
\end{aligned}
$$

where A, B, D, E are initial and final products and all rate constants are equal to 1.

Q8. Provide a full phase plane analysis for the heartbeat equations given by

$$
\begin{aligned}
\epsilon \frac{dx}{dt} &= -(x^3 - Tx + b), \quad T > 0 \\
\frac{db}{dt} &= (x - x_0) + (x_0 - x_1)u,
\end{aligned}
$$

by appropriately defining the control variable u associated with the pacemaker, x is the muscle fibre length, b is the chemical control, $\epsilon > 0$, (x_0, b_0) is a rest state and x_1 corresponds to systolic state.

Q9. State Poincaré-Bendixson Theorem. Verify the existence/nonexistence of the limit cycle in the following dynamical system:

$$
\begin{aligned}
\dot{x} &= x + y - x(x^2 + 2y^2) \\
\dot{x} &= -x + y - y(x^2 + 2y^2).
\end{aligned}
$$

Q10. For the following system find the fixed points, classify them, sketch the neighboring trajectories and try to fill the rest of the phase portrait:

$$
\begin{aligned}
\dot{x} &= xy - 1 \\
\dot{x} &= x - y^3.
\end{aligned}
$$

Q11. Plot the phase portrait and classify the fixed points of the linear system

$$
\begin{aligned}
\dot{x} &= -3x + 2y \\
\dot{y} &= x - 2y
\end{aligned}
$$

Also find the solution.

Q12. Plot the phase portrait and classify the fixed points of the linear system

$$
\begin{aligned}
\dot{x} &= y \\
\dot{y} &= 7x + y
\end{aligned}
$$

Also find the solution.

Q13. Show that the nonlinear conservation system

$$
\begin{aligned}
&\dot{x} = y \\
&\dot{x} = \mu \sin x - x, \quad \mu \geq 0.
\end{aligned}
$$

has one equilibrium point for $0 \leq \mu < 1$ and three for $\mu > 1$. Also discuss the nature of the equilibrium points. **Q14.**An ancestral DNA sequence of 40 bases was

CTAGGCTTACGATTACGAGGATCCAAATGGCACCAATGCT,

and in a descendent, it had mutated to

CTACGCTTACGACAACGAGGATCCGAATGGCACCATTGCT.

1. Give an initial base distribution vector, a frequency table and a Markov matrix to describe the mutation process.
2. By assuming that these sequences are simulated according to Jukes-Cantora Model, compute the Jukes-Cantora distance upto 4 decimal places.

Q15. Show that the iterating scheme

$$
x_{n+1} = 1 - \mu x_n(1 - x_n)
$$

has a stable fixed point $x_0 = 1$ for $\mu < 1$ and that $\mu = 1$ is a bifurcation point where the fixed point $x_0 = \dfrac{1}{\mu}$ appears. Show that period doubling occurs as soon as μ exceeds 3.

Q16. Discuss the Jukes-Cantor model of base substitution used for DNA sequence. How is it different from the Kimura 2-parameter and 3-parameter models?

Q17. Show that product of two Jukes-Cantor matrices is again a Jukes-Cantor matrix.

Q18. Calculate $d_{JC}(S_0, S_1)$ of the two 40-base sequences:

$S_0 : CTAGGCTTAGTCCCCTTGATCTAGCATCCCTAGCTAACGT$

$S_1 : AGGTTTAGACCCCTGTATCTGACATACCTCGCTAACTTCC$

Q19. Ancestral and descendent sequences of 400 bases were simulated according to the Kimura 2-parameter model with $\gamma = \beta/5$. A comparison of aligned sites gave the frequency data in following table:

$S_1 \backslash S_0$	A	G	C	T
A	92	15	2	2
G	13	84	4	4
C	0	1	77	16
T	4	2	14	70

1. Compute the Jukes-Cantor distance to 10 decimal digits, showing all steps.
2. Compute the Kimura 2-parameter distance to 10 decimal digits, showing all steps.
3. Which of these is likely to be a better estimate of the number of substitutions per site that actually occurred? Explain.

Q20.

1. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 3 taxa.
2. Draw the three topologically distinct rooted bifurcating trees that could describe the relationship between 3 taxa.

Q21.

1. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 4 taxa.
2. Draw the three topologically distinct rooted bifurcating trees that could describe the relationship between 4 taxa.

Q22.

1. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 5 taxa.
2. Draw the three topologically distinct rooted bifurcating trees that could describe the relationship between 5 taxa.

Q23. Suppose four sequences $S_1, S_2, S_3,$ and S_4 of DNA are separated by phylogenetic distances as in following table:

	S_1	S_2	S_3	S_4
S_1		1.2	0.9	1.7
S_2			1.1	1.9
S_3				1.6

Construct a rooted tree showing the relationships between $S_1, S_2, S_3,$ and S_4 by UPGMA.

Q24. Suppose four sequences $S_1, S_2, S_3,$ and S_4 of DNA are separated by phylogenetic distances as in following table:

	S_1	S_2	S_3	S_4
S_1		.82	0.23	.44
S_2			.54	.97
S_3				.37

Construct a rooted tree showing the relationships between $S_1, S_2, S_3,$ and S_4 by Neighbor Joining Method.