

# DEPARTMENT OF MATHEMATICS

Mathematics has a dual nature: It is a science and way of thinking, with its own language designed for logical discourse, and it also provides unique approaches to describing and understanding reality. Much of modern life rests on intellectual and scientific developments that are directed by mathematical equations and algorithms: space flight, computers, the Internet, weather modelling, security codes, and a lot of others. Students would familiarity with two primary aspects of mathematical thinking after graduating in this course.

The first aspect is mathematics as a body of knowledge. It is concerned with such issues as enumeration and computation, quantifying change, geometrical figures, shape, and symmetry. It deals with these topics via precise, unambiguous symbolic language. Students would understand some of the aesthetically beautiful ideas and their history that have implications so powerful that science and technology would be impossible without this underpinning—selected from topics such as number theory, geometric analysis, calculus, probability and statistics, Combinatorics, and symbolic logic, among others.

The second aspect of mathematical thinking is its broad applicability, its "unreasonable effectiveness" in the natural, biological and engineering sciences, as well in many of the social sciences and psychology. The essential concept is "mathematical modelling." Using mathematical ideas many problems that arise in the everyday world can be abstracted and expressed as mathematical problems. The solutions, often obtained via scientific computation, are then applied to the original problem, and their conformance to reality checked. These elegant solutions to applied problems are necessary for a deeper understanding of the forces that continuously transform our world. The hands-on experiment of mathematical software *MATHEMATICA*, *MATLAB*, *R-Software* and *TORA* will help gain a clear understanding of the subject.

## Course offered: B.Sc. (Honours) Mathematics

Link to Syllabus:

[http://www.du.ac.in/du/uploads/Syllabus\\_2015/B.Sc.%20Hons.%20Mathematics.pdf](http://www.du.ac.in/du/uploads/Syllabus_2015/B.Sc.%20Hons.%20Mathematics.pdf)

# Course Outcome:

After completing graduation in mathematics, there is a wide array of career prospects in mathematics field. All the highest paying jobs are directly or indirectly related with mathematics. Mathematics jobs are available in both government as well as private organizations. In the government sector, mathematics graduates may be employed in government departments, semi-governments, PSUs, research organizations, technical branches, banking sectors, colleges and universities. Besides academic jobs, trained mathematicians are also engaged at good remuneration and package in Indian Space Research Organisation (ISRO), Defence Research and Development Organisation (DRDO), National Aeronautics and Space Administration (NAL) and Society for Electronic Transaction and Security (SETS). Financial Mathematics is another booming area where student can get one of the best packages in the industry. IT giants like IBM and Microsoft hire candidates as scientists where salary is beyond your expectation.

## A. Expected Learning Outcomes for Core Courses

### 1. C 1: Calculus

Calculus is essential for many areas of science and engineering. Both make heavy use of mathematical functions to describe and predict physical phenomena that are subject to continuous change, and this requires the use of calculus. This course provides a comprehensive survey of differential and integral calculus concepts, including limits, derivative and integral computation, linearization, Riemann sums, the fundamental theorem of calculus, and differential equations. Content is presented in 5 units and covers various applications, including graph analysis, linear motion, average value, area, volume, and growth and decay models. In this course students use an online textbook, which supplements the instruction they receive and provides additional opportunities to practice using the content they've learned. Students will learn to use an embedded graphing calculator (**CASIO 991 ES**) and Mathematical Software **MATHEMATICA** for their Practical understanding of the topic.

### 2. C 2 :Algebra

Becoming an algebra expert opens the doors to some of today's most trendy (and well-paid) careers. From computer science to medicine, algebra serves as a foundational skill. Understanding algebra also puts students on track for college success, no matter what major they choose. The content of the course is well defined and give a glimpse of Complex

Number, Linear Algebra, Analytical Geometry and Applied Algebra. Content is presented in 5 units and covers various applications, including Polar representation of complex numbers,  $n$ th roots of unity, De Moivre's theorem for rational indices and its applications. Equivalence relations, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, Systems of linear equations, row reduction and echelon forms, applications of linear systems, linear independence. Subspaces of  $\mathbb{R}^n$ , dimension, rank, Eigen values, Eigen Vectors and Characteristic Equation of a matrix. The scope of this learning experience is to study basic mathematical concepts required for adult learners at a college or university level. Learners will become proficient at solving linear and quadratic equations, graphing linear and quadratic equations, functions, exponents and logarithms, and applying these concepts to real world problems.

### *3. C 3: Real Analysis*

The course of Real Analysis is designed in such a way that students should understand mathematical analysis dealing with the real numbers and real valued functions of a real variable. In particular, it deals with the analytic properties of real functions and sequences, including convergence and limits of sequences of real numbers, the calculus of the real numbers, and continuity, smoothness and related properties of real-valued functions. The curriculum is divided into five units.

### *4. C 4: Differential Equation*

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology.

The content covered in the widest sense in four units: ordinary and partial differential equations, functional and abstract differential equations, dynamical model including predator-prey model and its analysis, competing species and its analysis, epidemic model of influenza and its analysis, battle model and its analysis, integro-differential equations etc. The topics include the study of asymptotic behavior, stability, oscillations, periodic solutions, bifurcations and applications to ecology, population, biology, and engineering sciences. The student will design and analyse the

outcomes of above model and predict the result through mathematical software Mathematica/Matlab.

#### 5. *C 5 :Theory of Real Functions*

The course covered in 7units that deals with, Limits of functions (epsilon-delta approach), Continuity of functions ,Uniform continuity, Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem Taylor's series &Maclaurin's series. The students will be able to understand the behavior of sequence and series, its convergence and divergence property and the application of it in real world.

#### 6. *C 6: Group Theory-I*

The subject gives the idea about the general structure of elements occur in our society. The types of group they formed and their properties, including Dihedral groups, permutation groups and quaternion groups , Subgroups, centralizer, normalizer, center of a group, product of two subgroups. Properties of cyclic groups, classification of subgroups of cyclic groups, alternating group, etc. The topics also covered Lagrange's theorem and consequences including Fermat's Little theorem, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups. It also discuss the homeomorphisms, properties of homeomorphisms, properties of isomorphism. The entire topic is covered in 3units

#### 7. *C 7: Multivariate Calculus (including practical)*

The students get to learn the advanced calculus that described in 4 units, which includes homeomorphisms, properties of homeomorphisms, Cayley's theorem, properties of isomorphism, Lagrange multipliers, constrained optimization problems, vector field, divergence and curl Double integration over rectangular region, double integration over nonrectangular region. Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co-ordinates. Line integrals, Applications of line integrals: Mass and Work. Fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stokes' theorem, The Divergence theorem are also being taught in the subject.

#### 8. *C8: Partial Differential Equations (including practical)*

The subject emphasize on the use of PDEs whenever multivariate functions occur in mathematical modeling. The student get hand on experience on mathematical software about mathematical modeling of vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws and Burger's equations and able to analyse the use and application in physical, biological and in engineering sciences. The theoretical knowledge of classification of second order PDE, reduction to canonical forms, equations with constant coefficients, general solution. finite strings with fixed ends, non-homogeneous wave equation, Riemann problem, Goursat problem, spherical and cylindrical wave equation, vibrating string problem, existence and uniqueness of solution of vibrating string problem, heat conduction problem, existence and uniqueness of solution of heat conduction problem, Laplace and beam equation, non-homogeneous problem is also being taught.

#### *9. C9: Riemann Integration & Series of functions*

This gives students to understand the classical approach of integration using Riemannian sum. The topic covered in five units, which includes Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums; Convergence of Beta and Gamma functions, Limit superior and Limit inferior. The subject also describes the Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem, which is of great help in higher mathematics.

#### *10. C10: Ring Theory & Linear Algebra-I*

The classical approach to understand the higher mathematics is prescribed in this topic through ring theory. The subject emphasis on rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homeomorphisms, its properties, Isomorphism theorems. In the second part it provide information about higher Linear Algebra, that includes Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations.

### *11. C11: Metric Spaces*

This topic covered the analysis of pure mathematics, which is prescribed in five units, Metric spaces Continuous mappings, sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mappings, Banach Fixed point Theorem. Connectedness, Compactness are the area to be covered in this subject.

### *12. C12: Group Theory-II*

This section continues with idea of pure mathematics through classical approach. It deals with automorphism, inner automorphism, Fundamental Theorem of finite abelian groups, Group actions, stabilizers and kernels, permutation representation associated with a given group action. It also includes applications of group actions: Generalized Cayley's theorem, Index theorem. Groups acting on themselves by conjugation, class equation and consequences, conjugacy in  $S_n$ ,  $p$ -groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of  $A_n$  for  $n \geq 5$ , non-simplicity tests.

### *13. C13: Complex Analysis*

The classical complex analysis is discussed in this section, which is covered in six units. Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions. c-r equation, Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Laurent series and its examples, absolute and uniform convergence of power series, is covered under this topic. Students also get a hand on experience of the above properties through mathematical software.

### *14. C14: Ring Theory and Linear Algebra-II*

Higher order mathematics is continue in this section, which provides information about Laurent series, absolute and uniform convergence of power series, uniqueness Bessel's inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.

## **B. Discipline Specific Electives**

### I. DSE: Cryptography and Network Security

Cryptography and encryption are most critical components of network security. Cryptography can be used to implement confidentiality, integrity, authentication, and nonrepudiation. Network security is concerned with the protection of network resources against alteration, destruction and unauthorized use. The course contains covers the topic required to understand the importance of Network Security using Cryptography. The various cryptographic methods discussed are cipher model, Classical encryption techniques- Substitution and transposition ciphers, caesar cipher, Playfair cipher. Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard, Chinese Remainder theorem, Advanced Encryption Standard(AES), Stream ciphers, RSA algorithm and security of RSA, elliptic curve cryptography, cryptographic Hash functions, Secure Hash algorithm, SHA-3. The learner will understand the safety and security of communication and main goal is user authentication, data authentication such as integrity and authentication, non-repudiation of origin, and confidentiality. The topic also covers digital signature, elgamal signature, digital signature standards, digital signature algorithm and E-mail security: Pretty Good Privacy (PGP).

### II. DSE: Discrete Mathematics

Computer Science, Graph theory and Boolean algebra are based on discrete mathematics. The content provides deep information about properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, switching circuits, basic properties of graphs, pseudo graphs, complete graphs, bipartite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm covered under three units.

### III. DSE: Probability theory & Statistics

Probability and statistics were the only well-founded theories of uncertainty for a long time. Statistics is about gaining information from sets of data. Sometimes you want to represent a lot of complicated information from a large data set in a way that is easily understood. This is called *descriptive statistics*. Statistics is intimately linked to probability theory. students can use statistics to work out the probability, the chance that a certain event will occur. This course provides a comprehensive survey of *descriptive statistics* and *Probability* concepts, including

discrete and continuous random variables, cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, marginal and conditional distributions, expectation. Content is presented in 3 units and covers various applications, including Chebyshev's inequality, Central Limit theorem, Markov Chains, Chapman-Kolmogorov equations, classification of states etc. Students will learn to use an embedded graphing calculator (CASIO 991 ES) and Mathematical/Statistical Software MS-EXCEL AND SPSS for their Practical understanding of the topic.

#### IV. DSE: Linear Programming and Theory of Games

Life is full of conflicts and competition. The examples of conflicts include parlour games, military battles, political campaigns, advertising and marketing campaigns by competing business firms etc. There is a great scope for economists, statisticians, administrators and the technicians working as a team to solve problems of defense by using the OR approach. Besides this, OR is useful in the various other important fields like: 1. Agriculture 2. Finance 3. Industry 4. Marketing and Personnel management 5. Production Management 6. Research and Development etc. The content is divided in four units and covers the topic including Introduction to linear programming problem, simplex method, optimality and unboundedness, the simplex algorithm, two-phase method, Big-M method and their comparison. Duality, economic interpretation of the dual. Transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem and Game theory: formulation of two person zero sum games, mixed strategies, graphical solution procedure, and linear programming solution of games.

#### V. DSE : Numerical Analysis

Problems of the real world have measurement errors that propagate step by step and started dominating the desired result. The iterative process in Numerical analysis aims to reduce the error as much as possible through various technique/algorithm. Most simulators of physical phenomena Maxwell's equations (electromagnetic phenomena), Navier-Stokes equations (fluid mechanics), Schrödinger's equation (quantum mechanics), Fourier equations (heat conduction), etc are using numerical methods rather than closed-form solutions. The prescribed course curriculum is divided into four units and help students, 1) to get to know how fast errors cause problems 2) to find better algorithms that cause less errors. They will learn the different method



to generate solutions of mathematical modelling using Newton-Raphson Method, Secant Method, Iterative Method, Gauss-Siedel Method, trapezoidal, Newton-cotes Method, Runge-Kutta Method etc. that minimizes the error while computing. Students will learn to use an embedded graphing calculator (**CASIO 991 ES**) and Mathematical Software **TORA** for their Practical understanding of the topic.

### C. Skill Enhancement Courses

#### I. SEC\_1 [LATEX]

The Skill Enhancement Courses are skill-based and are aimed at providing hands-on-training, competencies, skills, etc. through *the tools of modern Mathematics*. One such important tool is LATEX, which is an easy-to-use version of TEX designed by Leslie Lamport. LATEX files are text files, which we can edit with simple text editors such as Notepad or Notepad++. There are dedicated editor for L ATEX files.

While it is possible to create beautiful mathematical documents with other software, no other program has the ubiquity of LATEX for mathematical writing. It can be used to type assignments, research articles and books involving equations, in more convenient ways than any other available software. This course provides comprehensive information including graphic design using PS-tricks, beamer presentation formats etc. The hands on training on LATEX will open the door for the students to be a freelancer or can associate themselves with publishing companies, as many of the reputed publishing companies require LATEX expert and they also outsource the assignment for freelancers.

#### II. SEC\_2 [COMPUTER ALGEBRA SYSTEM]

A computer algebra system is a program with which students can perform calculations, evaluate functions, create graphics, and develop their own programs. The key feature of computer algebra systems is the ability to manipulate expressions symbolically. Typical manipulations possible in a CAS include simplifying expressions, factoring, taking derivatives, computing integrals (symbolically and numerically), and solving systems of equations. The course curriculum is designed in such a way that enhance their skill in Mathematical/Statistical Software like, **MATHEMATICA**, created by Stephen Wolfram, **MAPLE**, created by Waterloo Maple, **MAXIMA**, **R-software** and **MATLAB**. The exposure of such software can help them to work

as an independent analyst, financial assistant, survey analyst or can be easily associate themselves with other firms.

- III. SEC\_3 [R-Software]
- IV. SEC\_4 [Transportation and Game Theory ]

## **GENERIC ELECTIVES OFFERED BY DEPARTMENT OF MATHEMATICS TO STUDENTS OF OTHER COURSES**

- I. GE\_1a: Calculus
- II. GE\_1b: Analytic Geometry and Theory of Equations
- III. GE\_2a: Linear Algebra
- IV. GE\_2b: Discrete Mathematics
- V. GE\_3a: Differential Equation
- VI. GE\_3b: Linear Programming and Theory of Games
- VII. GE\_4a: Numerical Methods
- VIII. GE\_4B: Elements of Analysis

### Mathematics Papers for Students of BA(Program)

- I. Paper \_1: Calculus
- II. Paper \_2: Algebra
- III. Paper \_3: Analytic Geometry and Applied Algebra :-The subject aims to impart analytical knowledge of geometry and application of network analysis in real world problem to the students of BA(P). The content is prescribed in three units and covers various application of conics, including graphing of conics, 2-D &3-D graphing of sphere, cylinders, mathematical modelling of Matching Job, Network reliability, Street Surveillance, Scheduling Meetings, Spelling Checker etc.
- IV. Paper \_4: Analysis
- V. Paper \_5: Differential Equations
- VI. Paper \_1: Numerical Analysis